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Measurement of the Newton Gravitational Constant *G*

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A novel method for measuring the gravitational constant G utilizes a modified Cavendish arrangement in which the gravitational interaction between a large mass system and a small mass system produces a torque on the latter. A rotary table carrying the large masses is servoed to constant relative geometry with respect to the small mass system. The angular acceleration of the table and the other appropriate parameters are measured. Recently [Phys. Rev. Letters **23**, 655 (1969)] a slightly improved value of G was reported. Further improvements in the method and noise reduction are described and the ultimate potential accuracy of the method is discussed.

Key words: Gravitation; Newton gravitational constant; universal gravitational constant.

1. Introduction

Gravitational interaction possesses two unique properties; its universality and its extreme weakness. Both of these features contribute to the difficulty of measuring the gravitational constant G which occurs in Newton's law of gravitation

$$F=G(m_1m_2/d^2) \tag{1}$$

where F is the force of attraction between any two particles of matter in the universe having masses of m_1 and m_2, and d is the distance between the particles. The relatively small size of masses which can be used in practical laboratory experiments to determine the gravitational constant, essentially requires the absolute measurements of very minute forces or torques. Furthermore, these forces or torques must be measured in the presence of disturbing forces and force gradients which result from the asymmetry of the mass distribution around the experiment, i.e., the test masses interact not only with each other but also with every other mass in the universe, for it has not been possible to isolate or shield them from these perturbing forces.

Critical reviews [1, 2] have been given of a number of most ingenious experiments devised and carried out by highly talented and skillful workers for measuring G. However, it is evident that there still exists an urgent need for determining with as much accuracy as possible both the absolute value of G and a possible variation of G with time and other factors.

A few years ago the authors [3] proposed a new experimental method for determining G and some preliminary results have been briefly reported [4, 5]. This paper will present additional preliminary results as well as a more detailed description of the method and the apparatus used. The principle of the method is illustrated in figure 1.

Two large spherical masses (tungsten spheres) are mounted on a rotary table which can be driven about its axis of rotation by a specially designed electric motor. Also mounted from the same rotary table is a gas-tight chamber in which a small horizontal cylinder is suspended by means of a quartz torsion fiber fastened to the top of the cylinder and hanging in the axis of the chamber. This small horizontal cylinder is commonly called the small mass system as contrasted to the large spheres which are referred to as the large mass system.

The gravitational interaction between the two mass systems tends to cause the small mass system to deflect so as to bring its axial center line in alignment with the line connecting the centers of the two large spheres. This changes the angle, θ. However, a beam of light from a source mounted on the rotary table is reflected from a mirror mounted on the small mass system near the axis of the quartz fiber and falls on a photodiode, also mounted on the rotary table. Thus the light beam generates an angle β.

As θ begins to change due to the gravitational interaction, so does β. The photodiode senses very minute changes in β, and this "error" signal is used

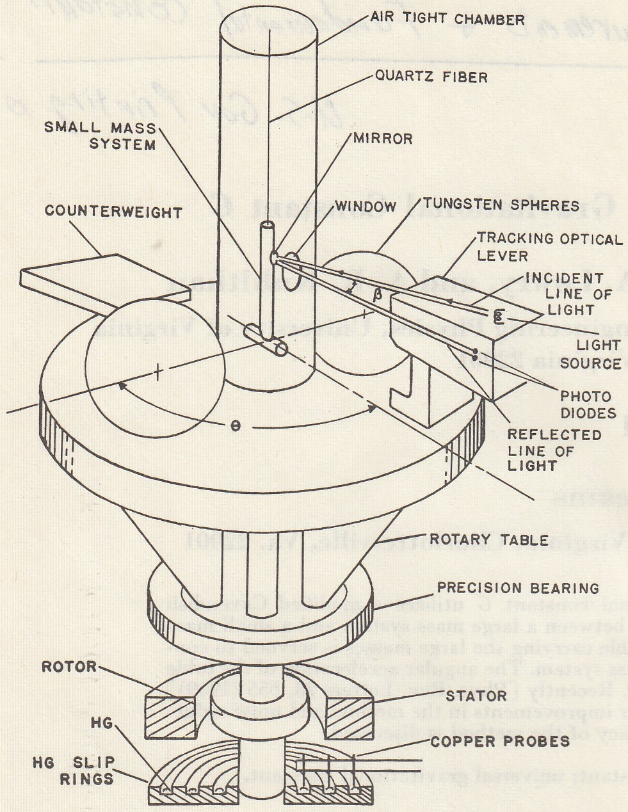

FIGURE 1. *Schematic drawing of experimental apparatus.*

to drive the motor which rotates the table so as to maintain β (and hence θ) constant. Thus with the angular separation, θ, remaining constant, the small mass system experiences a constant torque, which in turn causes a constant angular acceleration of the rotary table. This acceleration can be determined very accurately by measuring the period of the rotating table, and will be shown to be a direct measure of G.

This method possesses two novel features which contribute to the potential for improved accuracy. First, the interaction force of the two masses is manifested in an acceleration (change of rotational velocity) rather than a deflection. The effect of the interaction is cumulative and can be integrated over a long period of time, thus improving precision. Second, the two mass systems rotate about an axis many times during a measurement, and hence the effects of gravitational fields or field gradients due to extraneous masses are effectively cancelled except for higher order effects.

2. Theoretical Analysis of the Gravitational Torque

The analysis of the gravitational torque on the small mass system in figure 1 exerted by the spheres can be separated into two distinct parts. In the first part the gravitational potential at a point outside the cylinder is calculated and in the second part the potential gradient is used to determine the torque and acceleration the small mass system will experience.

Referring to figure 2, the gravitational potential ϕ on the z axis for $z>C$ of a ring of mass m, radius r and located at $z=C\cos\alpha$ is

$$\phi_{\text{ring}}(P')=\frac{mG}{(R^2+C^2-2RC\cos\alpha)^{1/2}}. \qquad (2)$$

This can be expanded to Legendre polynomials to give

$$\phi_{\text{ring}}(P')=mG\sum_{l=0}^{\infty}\frac{C^l}{R^{l+1}}P_l(\cos\alpha). \qquad (3)$$

The gravitational potential at any point in space is obtained by multiplying each member of this series by $P_l(\cos\theta)$.

$$\phi_{\text{ring}}(R,\theta)=mG\sum_{l=0}^{\infty}\frac{C^l}{R^{l+1}}P_l(\cos\alpha)P_l(\cos\theta). \qquad (4)$$

The potential for a cylinder is obtained by writing this equation in differential form and integrating over r and z.

$$\phi(R,\theta)=\frac{2\pi\rho G}{R}\sum_{l=0}^{\infty}\frac{P_l(\cos\theta)}{R^l}$$

$$\times\int_{-L/2}^{+L/2}\int_0^a C^l P_l(\cos\alpha)\,r\,dr\,dz \qquad (5)$$

where ρ is the density of the cylinder.

Performing the integrations in (5) using Rodrigues' formula for $P_l(\cos\alpha)$ and expanding it by means of the binomial expansion yields,

$$\phi(R,\theta)=\frac{mG}{R}\sum_{l=0}^{\infty}\sum_{k=0}^{l/2}\sum_{j=1}^{k+1}A_{jkl}\left(\frac{2a}{L}\right)^{2(j-1)}\left(\frac{L}{2R}\right)^{l}P_l(\cos\theta) \qquad (6)$$

where $m=\rho\pi a^2L$,

$$A_{jkl}=\frac{A_{kl}}{(k+1)(l-2j+3)}\binom{k+1}{j}, \qquad (7)$$

$$A_{kl}=\frac{(-1)^k}{2^l l!}\binom{l}{k}\frac{(2l-2k)!}{(l-2k)!}, \qquad (8)$$

$$\binom{l}{k}=\frac{l!}{k!(l-k)!}, \qquad (9)$$

and a, R, and L are as shown in figure 2. The potential is nonvanishing only for even values of l.

This potential may now be used to determine the magnitude of the force on a point mass, M, located at the point P,

$$\bar{F}=M\nabla\phi. \qquad (10)$$

This is the same force that M exerts on the cylindrical mass by Newton's third law. The component of force perpendicular to R is the only one

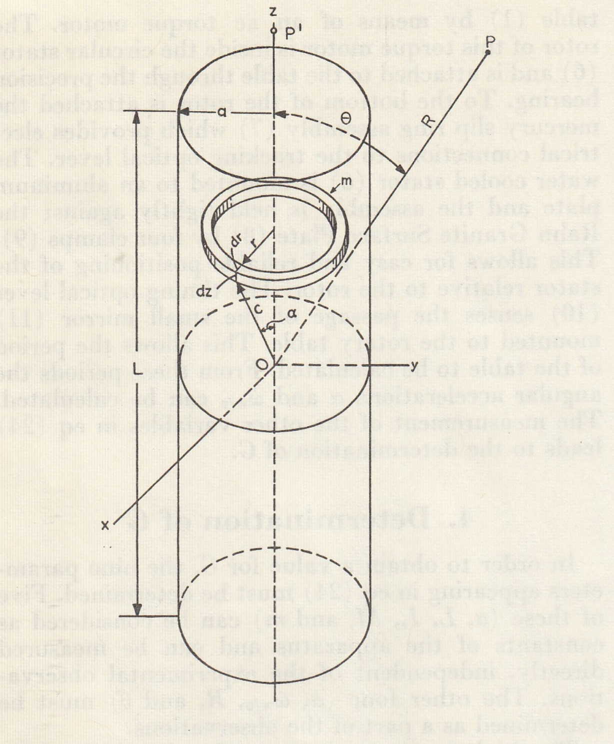

FIGURE 2. *Geometry for discussion of gravitational torque on a cylinder.*

which gives rise to a torque about 0. In spherical coordinates, this component is given by

$$F_\theta = (M/R)\,(\partial\phi/\partial\theta)\,\hat{\theta}. \qquad (11)$$

When two equal point masses are placed diametrically opposite each other at (R, θ) and $(R, \theta+\pi)$ a pure torque about the x-axis results, given by

$$T = 2M(\partial\phi/\partial\theta). \qquad (12)$$

Applying this equation to eq (6) yields

$$T = \frac{2mMG}{R} \sum_{l=0}^{\infty} \sum_{k=0}^{l/2} \sum_{j=1}^{k+1} A_{jkl} \left(\frac{2a}{L}\right)^{2(j-1)} \times \left(\frac{L}{2R}\right)^{l} \frac{\partial}{\partial\theta}\left[P_l(\cos\theta)\right]. \qquad (13)$$

The angular acceleration acting on the small mass system is

$$\dot{\omega} = T/I. \qquad (14a)$$

The stem holding the cylindrical mass is assumed axially symmetric about axis y (see fig. 1) and hence will not contribute to T but will contribute to I which is the moment of inertia of the torsional pendulum. With care, the contribution of the mirrors to T can be minimized and represents a negligible source of error. Their contribution to I is readily calculable and was taken into account.

Hence the angular acceleration acting on the torsional pendulum is

$$\dot{\omega} = \frac{2mMG}{IR} \sum_{l=0}^{\infty} \sum_{k=0}^{l/2} \sum_{j=1}^{k+1} A_{jkl} \left(\frac{2a}{L}\right)^{2(j-1)} \times \left(\frac{L}{2R}\right)^{l} \frac{\partial}{\partial\theta}\left[P_l(\cos\theta)\right]. \qquad (14b)$$

Evaluating the $l = 2$, 4, and 6 poles yield,

$$\dot{\omega}_2 = (3mMG/IR^3)\left[\tfrac{1}{3} - \tfrac{1}{4}(2a/L)^2\right](L/2)^2 \sin 2\theta \qquad (15)$$

$$\dot{\omega}_4 = \dot{\omega}_2\, \frac{5}{6}\left(\frac{L}{2R}\right)^2 \frac{\left[\tfrac{1}{5} - \tfrac{1}{2}(2a/L)^2 + \tfrac{1}{8}(2a/L)^4\right]}{\left[\tfrac{1}{3} - \tfrac{1}{4}(2a/L)^2\right]} \times (7\cos^2\theta - 3) \qquad (16)$$

$$\dot{\omega}_6 = \frac{\dot{\omega}_2}{48}\left(\frac{L}{2R}\right)^4 \frac{\left[\tfrac{1}{7} - \tfrac{3}{4}(2a/L)^2 + \tfrac{5}{8}(2a/L)^4 - \tfrac{5}{64}(2a/L)^6\right]}{\left[\tfrac{1}{3} - \tfrac{1}{4}(2a/L)^2\right]} \times \left[1386\cos^4\theta - 1260\cos^2\theta + 210\right]. \qquad (17)$$

Of course, $\dot{\omega}$ is

$$\dot{\omega} = \dot{\omega}_2 + \dot{\omega}_4 + \dot{\omega}_6 + \cdots \qquad (18)$$

where the higher terms are negligible for the current measurement accuracy of $\dot{\omega}$.

Equation (18) is correct if the torsional constant, k, of the pendulum is zero. If $k \neq 0$ then the measured angular acceleration, α, is

$$\alpha = \dot{\omega} \pm \dot{\omega}_{w/o} \qquad (19)$$

where $\dot{\omega}_{w/o}$ is the angular acceleration without (w/o) the spheres on the rotary table. The plus sign of course applying to $\dot{\omega}$ and $\dot{\omega}_{w/o}$ being in the same direction.

An expression for G is easily obtained. From eq (19),

$$\dot{\omega} = \alpha \pm \dot{\omega}_{w/o}. \qquad (20)$$

Using eq (15) through (17), eq (18) may be written as,

$$\dot{\omega} = GA(1 + B + C + \cdots) \qquad (21)$$

where A, B, and C follow clearly from (15) through (17).

Since I is actually the moment of inertia of the small mass system (the moment of inertia of the fiber is negligible) which consists of the horizontal cylinder and the vertical stem,

$$I = I_r + I_s. \qquad (22)$$

This may be written as

$$I = m(L/2)^2\left[\tfrac{1}{3} + \tfrac{1}{4}(2a/L)^2 + (2/L)^2(I_s/m)\right]. \qquad (23)$$

Hence an expression for G, is

$$G = (\alpha \pm \dot{\omega}_{w/o})/A(1 + B + C \cdots) \qquad (24)$$

where

$$A = \frac{3M}{R^3}\, \frac{\left[\tfrac{1}{3} - \tfrac{1}{4}(2a/L)^2\right]\sin 2\theta}{\left[\tfrac{1}{3} + \tfrac{1}{4}(2a/L)^2 + (2/L)^2(I_s/m)\right]}, \qquad (25)$$

$$B = \frac{5}{6}\left(\frac{L}{2R}\right)^2 \frac{\left[\tfrac{1}{5} - \tfrac{1}{2}(2a/L)^2 + \tfrac{1}{8}(2a/L)^4\right]}{\left[\tfrac{1}{3} - \tfrac{1}{4}(2a/L)^2\right]}\,(7\cos^2\theta - 3), \qquad (26)$$

and

$$C=\frac{1}{48}\left(\frac{L}{2R}\right)^4 \frac{[\frac{1}{7}-\frac{3}{4}(2a/L)^2+\frac{5}{8}(2a/L)^4-\frac{5}{64}(2a/L)^6]}{[\frac{1}{3}-\frac{1}{4}(2a/L)^2]} \times[1386 \cos^4\theta-1260 \cos^2\theta+210]. \quad (27)$$

The experimental problem is to determine α, $\dot{\omega}_{w/o}$, R, M, a, L, I_s, m, and θ as accurately as possible. Then G can be calculated from eq (24). Additional terms can be calculated if necessary, but they do not contribute to the present level of precision.

3. Description of Apparatus

Figure 3 is a photograph of the experimental apparatus which is mounted inside a clean temperature controlled (0.01 °C) room. Two concrete block columns mounted on a concrete base (isolated from the building) support the experimental apparatus. The rotary table (1) and precision air bearing (2) support a gas tight chamber (3) which encloses a torsional pendulum immersed in helium gas to reduce any temperature gradients. The tungsten spheres (4) are placed on their stands so that their radial position relative to the center of the chamber is accurately known. The tracking optical lever (5) is of a type due to Jones and Richards [6], and monitor any angular change of the torsion pendulum inside the chamber. A servo mechanism is designed to maintain a zero output from the sensing photodiodes of the optical lever. To do this it rotates the rotary table (1) by means of an ac torque motor. The rotor of this torque motor is inside the circular stator (6) and is attached to the table through the precision bearing. To the bottom of the rotor is attached the mercury slip ring assembly (7) which provides electrical connections to the tracking optical lever. The water cooled stator (6) is mounted to an aluminum plate and the assembly is held tightly against the Rahn Granite Surface Plate (8) by four clamps (9). This allows for easy and reliable positioning of the stator relative to the rotor. The timing optical lever (10) senses the passage of the small mirror (11) mounted to the rotary table. This allows the period of the table to be calculated. From these periods the angular accelerations α and $\dot{\omega}_{w/o}$ can be calculated. The measurement of the other variables in eq (24) leads to the determination of G.

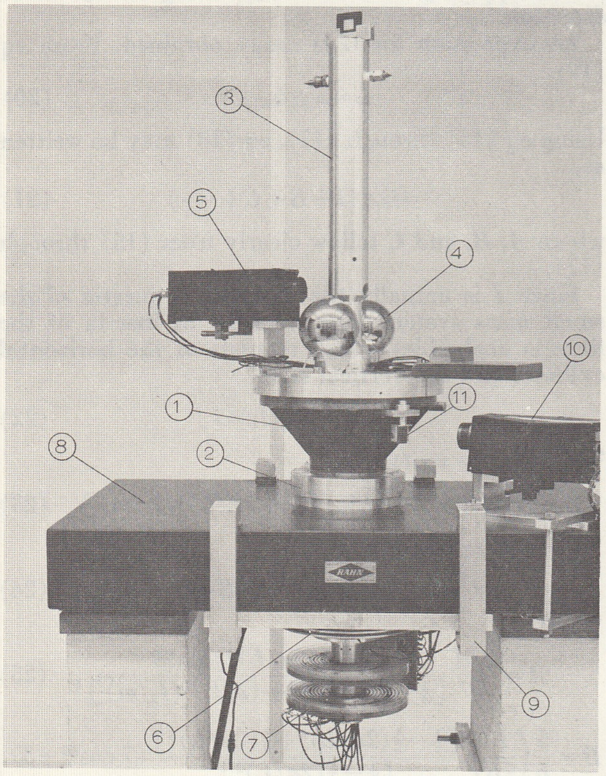

FIGURE 3. *Photograph of apparatus.*

4. Determination of *G*

In order to obtain a value for G, the nine parameters appearing in eq (24) must be determined. Five of these (a, L, I_s, M, and m) can be considered as constants of the apparatus and can be measured directly, independent of the experimental observations. The other four (α, $\dot{\omega}_{w/o}$, R, and θ) must be determined as a part of the observations.

The high-density tungsten spheres, which make up the large mass system (figs. 1 and 3) were specially made at the Y-12 plant of the Union Carbide Corporation Nuclear Division, Oak Ridge, Tenn. [7]. Each of the tungsten spheres had a mass of approximately 10 Kg and rested on a damped three point mount which in turn was supported by a common large quartz plate in order to minimize the effect of temperature fluctuations. The distance between the centers (fig. 1) is approximately 12 cm and was measured by standard methods using Johansson gauge blocks as references.

The small mass system consists of a carefully machined high purity oxygen free (diamagnetic) copper cylinder approximately 4 cm long (L) and 0.2 cm in diameter (a) fastened to a small accurately machined aluminum alloy rod which in turn was supported by a 25 μ quartz fiber 33 cm long hung from the top of the metal chamber containing helium as shown in figure 1. The mass of the copper cylinder m is about 4 g. The quartz fiber and aluminum alloy stem were carefully positioned in the vertical axis of rotation and the axis of the copper cylinder was in the horizontal plane containing the line between the center of the two tungsten spheres. The aluminum stem carried two small mirrors as shown in figure 1. The angle θ' between the axis of the copper cylinder and perpendicular to the tracking mirror on the aluminum stem is 44° 43′±0.5′.

The dimensions chosen for the small mass system were a compromise between feasible size and keeping the distance between the two spheres $2R$ as small as possible for maximum angular acceleration α. The precise physical dimensions of the mass systems are shown in table 1.

TABLE 1. *Mass systems*

Large masses—high density tungsten spheres		Sphere #1	Sphere #2
Mass	kg	10.489980±0.00007	10.490250±0.00007
Diameter	cm	10.165072	10.165108
Distance between center of mass and geometrical center	cm	4.610×10^{-4}	7.569×10^{-4}
Sphericity	cm	12×10^{-6}	12×10^{-6}
Small mass system—copper cylinder			
Length L	cm	3.9649±0.0004	
Diameter a	cm	0.19824±0.0004	
Mass m	g	4.0512±0.0001	
Moment of inertia of aluminum alloy system	g cm²	0.0408±0.0001	
Angle between the axis of copper cylinder and perpendicular to tracking mirror		44°43′±0.5′	

The angle θ between the longitudinal axis of the cylindrical rod of the small mass system and an imaginary line joining the centers of mass of the spheres not only must be determined but must be held constant during the experiment. As can be seen from eq (15), it is desirable to make this angle approximately 45° in order to maximize the angular acceleration $\dot{\omega}$ of the table. In effect, this will minimize the experimental error since when θ is zero or 90° there is no torque exerted on the small mass system. Consequently the torque must pass through a maximum when θ is varied from zero to 90°. It turns out that there is a relatively flat maximum near $\theta=45°$. In order to maintain θ constant, β is held constant by the servo system shown in the block diagram in figure (4). The details of this servo system are explained in the literature [8]. Experimental tests show that the tracking error of this system is approximately ±0.2 s of arc. The small mass system is suspended with the perpendicular to the tracking mirror also perpendicular to the line joining the centers of mass of the spheres. Consequently θ is very nearly equal to $90°-\theta'$ with small corrections determined from the position of the tracking optical lever [5]. The tracking optical lever is positioned so that the tracking angle corresponds as closely as possible to the neutral position of the small mass systems with the tungsten spheres removed. This minimizes the background angular acceleration of the table $\dot{\omega}_{w/o}$ due to twist in the quartz fiber.

The measurement of the angular acceleration α and $\dot{\omega}_{w/o}$ in eq (24) involves the determination of the period of the rotary table. Figure (5) is a block diagram of the timing system. When the table turns the attached mirror reflects the light back to the fixed optical lever producing a sharp voltage spike.

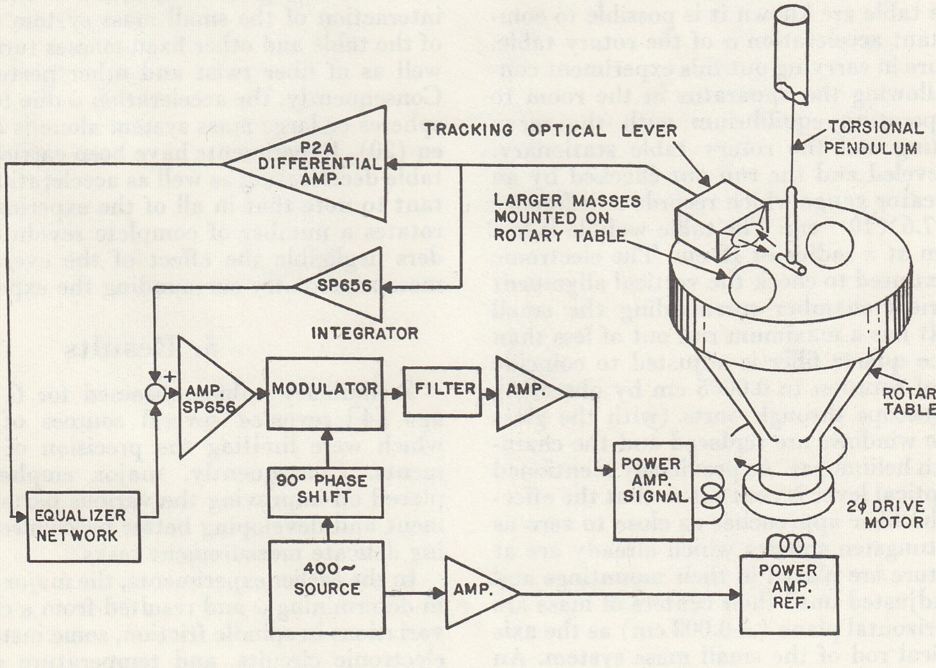

FIGURE 4. *Circuit block diagram for tracking loop.*

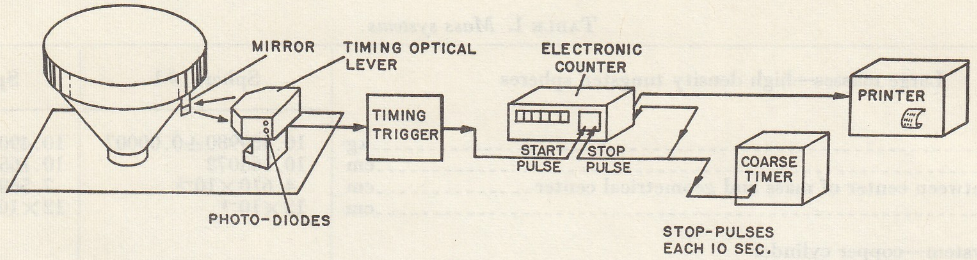

FIGURE 5. *Block diagram of timing system for measuring the period of each revolution.*

This activates the trigger when the output voltage exceeds a pre-set level. The timing trigger starts the counter (Hewlett-Packard Model 5245L) which measures the time interval from the start of the pulse until it receives a stop pulse from the counter's reference precision oscillator. The duration of this interval is relayed to a printer which records the interval in microseconds. Each 10 second interval thereafter from the reference oscillator is recorded. When the table makes a complete revolution another start pulse is produced by the same fixed optical lever which marks the beginning of a second revolution. This start pulse together with the next stop pulse from the reference oscillator will cause the electronic counter to print out the time interval between the two pulses in microseconds. The rotational period of the table is then obtained in microseconds. The reference oscillator in the counter has been checked against a 10 MHz broadcast signal from WWV and found to be 1 MHz to within 3 parts in 10^9. The error in this trigger system even at the maximum speed of rotation of the table is less than 1 μs. This introduces an error of less than 10^{-7} radians or a precision of at least 1 part in 10^7 for the period of rotation of the table. After the successive periods of rotation of the table are known it is possible to compute the constant acceleration α of the rotary table.

The procedure in carrying out this experiment consists in first allowing the apparatus in the room to come to temperature equilibrium with the servo system operating and the rotary table stationary. The table is leveled and the run out checked by an electronic indicator gauge which records a difference in position of 7.6×10^{-6} cm. The table wobble is less than 0.0005 cm at a radius of 12 cm. The electronic indicator is next used to check the vertical alignment of the cylindrical chamber surrounding the small mass system. It has a maximum run out of less than 0.0006 cm. The quartz fiber is adjusted to coincide with the axis of rotation to 0.0005 cm by observing it with a microscope through ports (with the glass removed). The windows are replaced and the chamber is filled with helium gas. As previously mentioned the tracking optical lever is next set so that the effective twist in the fiber approaches as close to zero as possible. The tungsten spheres which already are at room temperature are placed in their mountings and their heights adjusted until their centers of mass are in the same horizontal plane (±0.002 cm) as the axis of the cylindrical rod of the small mass system. An analysis shows that the error introduced by this misalignment is negligible in the present experiment. The centers of mass of the two tungsten spheres are placed in a line through and perpendicular to the axis of rotation by use of a combination of an interferometer plate mounted on top of the chamber, modified optical lever and electronic indicator. The electronic indicator is then used to make the radial distances of the two spheres from the axis of rotation equal to within $\pm10^{-5}$ cm. The absolute value of R is determined by comparison with Johansson gauges by standard optical methods used in connection with the electronic indicator.

The tracking is then started and the acceleration of the table determined. When operating properly this angular acceleration of the table is constant. In fact, the constancy of this acceleration is a good measure of the noise in the system. After the acceleration (between 4 and 5×10^{-6} rad/s^2) takes place for several hours and the rotational speed of the table reaches between one and two revolutions per minute, the large tungsten spheres are carefully removed from the table and the tracking continued for several hours. The acceleration of the table with the spheres removed $\dot{\omega}_{w/o}$ gives a measure of the gravitational interaction of the small mass system with the mass of the table and other fixed masses turning with it as well as of fiber twist and other perturbing factors. Consequently, the acceleration $\dot{\omega}$ due to the tungsten spheres or large mass system alone is determined by eq (20). Experiments have been carried out with the table decelerating as well as accelerating. It is important to note that in all of the experiments the table rotates a number of complete revolutions; this renders negligible the effect of the ever present fixed mass asymmetry surrounding the experiment.

5. Results

Preliminary values obtained for G some months ago [4] revealed several sources of uncertainties which were limiting the precision of the measurements. Consequently, major emphasis has been placed on improving the various parts of the experiment and developing better procedures for performing delicate measurement tasks.

In the earlier experiments, the major error occurred in determining $\dot{\omega}$ and resulted from a combination of variations in spindle friction, some instabilities in the electronic circuits, and temperature gradients pro-

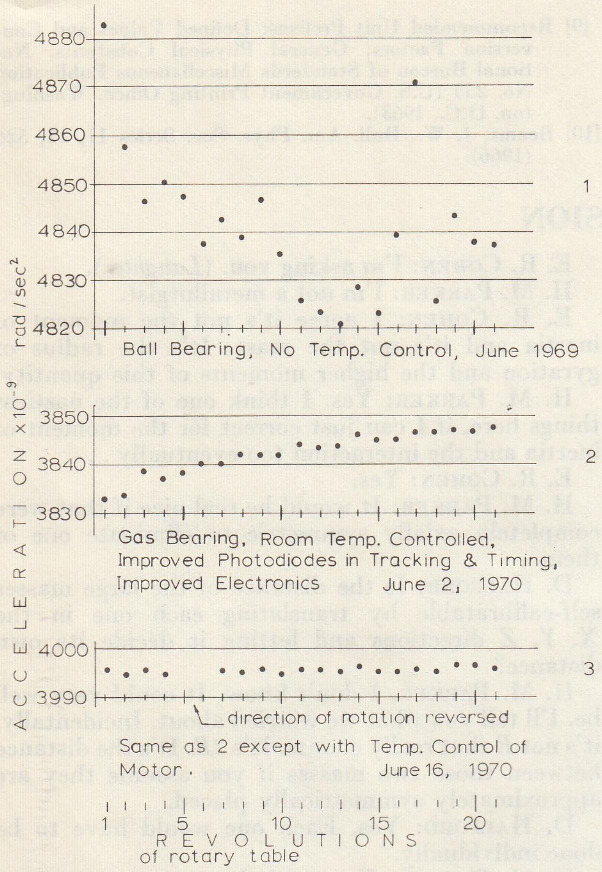

FIGURE 6. *Measurements of angular acceleration α.*

duced by the motor and bearing. Fortunately the trouble with the bearing has been solved by substituting a gas bearing for the precision ball bearing spindle used previously. Careful thermal isolation of the motor has reduced temperature variations and the electronic circuits have been much improved and stabilized. Very recent measurements of acceleration are shown in figure 6 and compared with earlier results. The acceleration now remains constant to the order of one part in 10^4. There is good reason to believe that this can be improved by at least a factor of ten in the near future. When the measurement of all the required physical dimensions of the system are completed a significant improvement in the accuracy of G should result. Before the recent improvements, our best value obtained with the present technique was $G=(6.674\pm0.012)\times10^{-11}$ Nm^2/kg^2 where 0.012 represents three standard deviations. This agrees very well with the value due to Heyl [9] of $G=(6.670\pm0.015)\times10^{-11}$ Nm^2/kg^2.

One limiting factor to the precision with which G can be determined with the present equipment is the precision with which the center of mass of the two tungsten spheres can be determined. These uncertainties produce an uncertainty in R which will produce an uncertainty in G of about 5 parts in 10^5. We are encouraged to believe that the method can be made to measure G with much greater accuracy than is possible with the present tungsten spheres.

Experiments [10] have shown that certain metals with intermediate to high specific gravities may have variations in density of less than one part in 10^7 per cm. These metals cannot be made into precise spheres because they will distort on the necessary supports under their own weight. However, it can be shown that they can be used in the form of rectangular blocks where the distortion will be greatly reduced. This of course complicates the theory but the problem is solvable by a computer.

It is difficult to estimate with reliability the ultimate precision with which G can be measured by this method. In agreement with the original conception of the experiment [3], it is still believed that magnetic suspension of the small mass system, rather than by quartz fiber, will provide the better ultimate accuracy. The primary reason is that almost surely the torsion constant of the small mass suspension system can be very significantly reduced and thus the required tracking tightness can be reduced. Also, it can be expected that the temperature sensitivity of the torsion constant can be reduced. Our experience has shown that many important improvements can be made in the present apparatus. With a completely new design of the rotary table and chamber with special attention to temperature and vibration isolation, certain changes in the servo motor drive system, improved metrology, substitution of the blocks of metal for the sintered tungsten spheres as well as of an accurately ground quartz cylinder in the small mass system, etc., it is not beyond conception that G can be measured to one part in 10^6. Also there is better reason to believe that variations in G of one part in 10^6 ultimately may be detected by this method.

6. Acknowledgments

We are especially indebted to Dr. James Senter and Dr. R. D. Rose for valuable contributions to the initial experiments. Also we wish to thank Dr. A. G. McNish and the members of the Metrology Section of the National Bureau of Standards for helpful advice, as well as Mr. R. Hibbs and Mr. E. W. Bailey of the Union Carbide Nuclear Company for assistance in obtaining the tungsten spheres. This work has been supported in part by a grant from the National Aeronautic and Space Administration.

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DISCUSSION

E. R. Cohen: I have some questions about how you really get one part in a million out of all of this. With the present system, I looked at your paper and tried to make some estimates. Going to the assembled masses I think would certainly help identify the center of mass properly. But in terms of the small mass, the attracted cylinder, don't you also have to worry about the density distribution in that? And I'm also worried about how many terms in your harmonic expansion are really needed to get a part in a million. I estimated, I think, at one time keeping up to the eighth order terms.

H. M. Parker: I have done up to tenth order. I don't think that's the problem. After all, that's just work. You can do that.

E. R. Cohen: Yes. But there is the question of the interaction, of how accurately you know one of the lower order terms compared to the higher order term that you're keeping.

H. M. Parker: Let me point out to you, though, that there is a great tendency for the mass in the small system to cancel out in the moment of inertia term and in the interaction term. If you consider your small mass system as a simple point dumbbell, then the mass doesn't appear at all. It's just the physical dimensions of it that count. I personally think that the thing that really counts here is the length of that cylinder and then—

E. R. Cohen: It's not the mass as much as the distribution, the density distribution.

H. M. Parker: Well, I think that one can show that even in that case that same tendency is there to some extent at least. I haven't shown that yet, but I think it's there. But I think that one can build systems like this, small things like this, with densities that are uniform to a part in 10^6, don't you?

E. R. Cohen: I'm asking you. (*Laughter*).

H. M. Parker: I'm not a metallurgist.

E. R. Cohen: I agree it's not the moment of inertia and it's not the mass. It's the radius of gyration and the higher moments of this quantity.

H. M. Parker: Yes. I think one of the nastiest things here, if I can just correct for the moment of inertia and the interaction too eventually . . .

E. R. Cohen: Yes.

H. M. Parker: It would be real nice if that were completely axially symmetric to eliminate one of them.

D. Halford: Is the distance of the large masses self-calibratable by translating each one in the X, Y, Z directions and letting it decide its own distance?

H. M. Parker. I don't know. It could very well be. I'll tell you what we worried about. Incidentally, it's not R that really counts. It's $2R$. It's the distance between those two masses if you assume they are approximately symmetrically placed.

D. Halford: Yes. Each one would have to be done individually.

H. M. Parker: Yes. And the way that we hope eventually to get at something about this non-uniformity of these tungsten spheres is to change their orientation. So far we have been careful to put them on the table the same way every time. But eventually we'll start turning them to different orientations and see if we get effectively different results. That will give us something about the inhomogeneity. There is likely to be fairly—I'm afraid too—large inhomogeneities in these sintered tungsten spheres. They were hard to mak . .